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TỐ HỢP VÀ HOÁN VỊ

PERMUTATION AND COMBINATION

Three pictures are to be hung in line on a wall. Indicating the different pictures by A, B and C, one order in which they can be hung is A, B, C and another is A, C, B.

Each of these arrangements is called a permutation of the three pictures (and there are further possible permutations).

i.e. a permutation is an ordered arrangement of a number of items.

Suppose, however, that seven pictures are available for hanging and only three of them can be displayed. This time a choice has first to be made. Representing the seven pictures by A, B, C, D,E,F and G, one possible choice of the three pictures for display is A, B, and C. Regardless of the order in which they are then hung this group of three is just one choice and is called combination. Thus A, B, C or A, C, B or B, A, C or B, C, A or C, A, B or C, B, A are six different permutations but only one combination .

i.e. a combination is an unordered selection of a number of items from a given set.

In general, n! represents the number nx(n-1)x(n-2)x…x2x1, i.e. n! means the product of all the integers from 1 to n inclusive.

Exercises :

1. In each of the following problems determine , without working out the answer, whether you are asked to find a number of permutations, or a number of combinations.

    - How many arrangements of the letters A, B, C are there ?
    - A team of six members is chosen from a group of eight. How many different teams can be selected ?
    - A person can take eight records to a desert island, chosen from his own collection of one hundred records. How many different sets

    of records could he choose ?
  - One red die and one green die are rolled (each number one to six). In how many ways can a total score of six be obtained ?

2. a) How many different combinations of six letters can be chosen from the letters A, B, C, D, E, F, G, H, if each letter is chosen only once ?

b) In how many ways can the eight letters be divided into two groups of six and two letters ?

1. A team of four children is to be selected from a class of twenty children , to compete in a quiz game. In how many ways can the team be chosen if :

   - any four children can be chosen
   - the four chosen must include the oldest in the class ?

2. A shop stocks ten different varieties of packet soup. In how many ways can a shopper buy three packets of soup if :

   - each packet is a different variety
   - two packet are the same variety

3. a) In how many ways can ten different books be divided into two groups of six and four books ?

b) In how many different hands of five cards can be dealt from a suit of thirteen cards?

1. A large box of biscuits contains nine different varieties. In how many ways can four biscuits be chosen if :

   - all four are different
   - two are the same and the others different
   - two each of two varieties are selected
   - there are the same and the fourth is different
   - all four the same

2. Find how many distinct numbers greater than 5000 and divisible by 3 can be formed from the digits 3,4,5,6 and 0, each digit being used at most once in any number.
3. A certain test consists of seven questions, to each of which a candidate must give one of three possible answers. According to the answer that he chooses, the candidate must score 1, 2, or 3 marks for each of the seven questions. In how many different ways can a candidate score exactly 18 marks in the test?
4. A tennis club is to select a team of three pairs, each pair consisting of a man and a woman, for a match. The team is to be chosen from 7 men and 5 women. In how many different ways can the three pairs be selected?
5. How many four digit odd numbers can be made from the set {5, 7, 8, 9}, no integer being used more than once?
6. How many numbers greater than 4000 can be made from the set {1,3,5,7}, if each integer can be used only once?
7. How many arrangements can be made of three letters chosen from PEAT if the first letter is a vowel and each arrangement contains three different letters?
8. How many three digit numbers can be made from the set of integers {1,2,3,4,5,6,7,8,9} if:

(a) the three digits are all different,

(b) the three digits are all the same,

(c) the number is greater than 600,

(d) all three digits are the same and the number is odd?

1. Three boxes each contain three identical balls. The first box has red balls in it, the second blue balls and the third green balls. In how many ways can three balls be arranged in a row if:

(a) the balls are of different colours,

(b) all three balls are of the same colour?

1. n red counters and m green counters are to be placed in a straight line. Find the number of different arrangements of the colours. A town has n streets running from south to north and m streets running from west to east. A man wishes to go from the extreme south-west intersection to the extreme north-east intersection, always moving either north or east along one of the streets. Find the number of different routes he can take.

hoán vị trên một hàng

ARRANGEMENT

The number of ways of arranging n unlike objects in a line is n! .

The number of ways of arranging in a line is n objects , of which p are alike , is .

The number of ways of arranging n unlike objects in a ring when clockwise and anticlockwise arrangements are different is (n-1)! .

The number of ways of arranging n unlike objects in a ring when clockwise and anticlockwise arrangements are the same, is .

e.g. The representatives of five countries attend a conference. In how many ways can they be seated at a round table ?

As they are seated at a round table, there is no first or last place to consider. What matters in this arrangement is where each one sits relative to the others, as no one seat is special.

EDCBA12345DCBAE12345CBAED12345BAEDC12345AEDCB12345Numbering the chairs 1 to 5, we can say that there are 5 ways of selecting the occupant of the first seat . 4 ways of selecting the occupant of the second seat … and so on. Thus there are 5x4x3x2x1 ways of arranging the representatives in the five seats . But this number includes the arrangements shown below :

i.e. for any one arrangement in the five seats , the representatives can be moved clockwise five times and still have the same people to the left and to the right of them . Therefore the 5x4x3x2x1 ways of arranging the five representatives in numbered seats is five times the number of ways of arranging them round a circular table , so there are ways, or 4x3x2x1 ways = (5-1)! in which the representatives can be seated at the round table.

e.g. In how many ways can five beads, chosen from eight different beads be threaded on to a ring ?

EDCBA12345DCBAE12345CBAED12345BAEDC12345AEDCB12345The number of ways of arranging five beads taken from eight different beads , in five numbered places is 8x7x6x5x4. Thus the number of ways of arranging five (from eight) beads in a circle is as :

But a ring can be turned over,

EDCBA12345BCDEA12345i.e.

and these have been counted as two separate arrangements. So the number of circular arrangements is twice the number of arrangements on a ring. Therefore the number of ways of threading five beads, from eight different beads , on a ring is .

1.

1. In how many ways can eight people be seated at a round table?
2. In how many ways can eight cows be placed in a circular milking parlour?

2.

1. How many different combinations of six letters can be chosen from the letters A, B, C, D, E, F, G, H, if each letter is chosen only once ?
2. In how many ways can the eight letters be divided into two groups of six and two letters?

3. In how many ways can five different books be arranged on a shelf ?

4. How many two digit numbers can be made from the set {2,3,4,5,6,7,8,9}, each number containing two different digits ?

5. In how many ways can six different shrubs be planted in a row?

6. In how many of the possible permutations of the letters of the word ADDING are the two D's :

1. together
2. separate

7. How many permutations of the letters of the word DEFEATED are there in which the E's are separated from each other ?

8. A box contains seven snooker balls , three of which are red , two black , one white and one green. In how many ways can three balls be chosen ?

9. Four books are taken from a shelf of eighteen books , of which six are paperback, and twelve are hardback. In how many of the possible combinations of four books is at least one a paperback ?

10. Find how many numbers between 10 and 300 can be made from the digits 1, 2, 3, if :

    1. each digit may be used only once,
    2. each digit may be used more than once ?

11. How many combinations of three letters taken from the letters A, A, B, B, C, C, D are there ?

12.

    1. Find the number of permutations of the letters of the word MATHEMATICS
    2. Find the number of permutations of four letters from the word MATHEMATICS
    3. How many of the permutations contain two pairs of letters that are the same ?

13. Find the number of ways in which twelve children can be divided into two groups of six if two particular boys must be in different groups ?

14. A box contains ten bricks , identical except for colour. Three bricks are red , two are white, two are yellow , two are blue and one is black. In how many ways can three distinguishable bricks be

    1. taken from the box
    2. arranged in a row ?

3. In how many of the arrangements in a row of all ten bricks are the three red bricks separated from each other ,
4. In how many of the arrangements in a row of all ten bricks are just two of the red bricks nect to each other ?

15. How many even numbers less than 500 can be made from the integers 1,2,3,4,5, each integer being used only once ?

16. In how many ways can three letters from the word GREEN be arranged in a row if at least one of the letters is E ?

bài tập tổ hợp

COUNTING AND ODERING TECHNIQUES

1.

TRACK MEET AMONG SCHOOLS A , B AND C

	First Place(5 points)	Second Place(3 points)	Third Place(1 point)
Event I	A		
Event II	A	B	
Event III		C	

A partially completed scorecard for a track meet is shown above . Schools A, B and C each entered one person in each of the three events and there were no ties. What is one possible total score for school C (Assume that all points are awarded in each event.)

2. In a stack of six cards , each card is labeled with a different integer 0 through 5. If two cards are selected at random without replacement , what is the probability that their sum will be 3 ?

3. A bag contains a number of pieces of candy of which 78 are red , 24 are brown, and the remainder are yellow . If the probability of selecting a

yellow piece of candy from this bag at random is 1/3 , how many yellow pieces of candy are in the bag ?

4. There are 120 red marbles and 80 blue marbles in a bag that contains 200 marbles . If only blue marbles are to be added to the bag so that the probability of randomly drawing a blue marble from the bag becomes 2/3 , how many blue marbles must be added to the bag ?

5. There are 3 roads from Plattsville to Ocean Heights and 4 roads from Ocean Heights to Bay Cove . If Martina drives from Plattsville Bay Cove and back , passes through Ocean Heights in both directions , and does not travel any road twice , how many different routes for the trip are possible.

6. A barrel contains only apples and oranges . There are twice as many apples as oranges . The apples are either red or yellow , and 4 times as many apples are red as are yellow. If one piece of fruit is to be drawn at random from the barrel , what is the probability that the piece drawn will be a yellow apple ?

7. A teacher is to be assigned to teach 5 different courses in 5 different class periods on Mondays. If exactly one course meets each period , how many different assignments of courses to these class periods are possible for Mondays ?

8.

Quiz Score	Number of Students Who Received That Score
0	3
1	6
2	7

3	4

A class of 20 students took a 3-question quiz. The table shows the possible scores on this quiz and the number of students who received each of these scores.If one of the students is picked at random , what is the probability that that student’s quiz score will be greater than 1 ?

9. Of the students in a certain homeroom , 9 are in the school play , 12 are in the orchestra , and 15 are in the choral group . If 5 students participate in exactly 2 of the 3 activities and all other students participate in only 1 activity, how many students are in the homeroom ?